Krylov Projection Framework for Fourier Model Reduction

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Abstract

This paper analyzes the Fourier model reduction (FMR) method from a rational Krylov projection framework and shows how the FMR reduced model, which has guaranteed stability and a global error bound, can be computed in a numerically efficient and robust manner. By monitoring the rank of the Krylov subspace that underlies the FMR model, the projection framework also provides an improved criterion for determining the number of Fourier coefficients that are needed, and hence the size of the resulting reduced-order model. The advantages of applying FMR in the rational Krylov projection framework are demonstrated on a simple example.

Key words: Model reduction; system order reduction

1 Introduction

Model reduction entails the systematic generation of cost-efficient representations of large-scale systems that result, for example, from discretization of partial differential equations (PDEs). Recent years have seen the development of several reduction methods that are computationally tractable for large-scale systems. These methods have been applied in many different settings with considerable success, including controls, fluid dynamics, structural dynamics, and circuit design.

Algorithms such as optimal Hankel model reduction [1,3,15,8] and balanced truncation [20,19] come with rigorous guarantees of quality and global error bounds on the resulting reduced models. These methods are usually referred to as singular value decomposition (SVD) or gramian-based model reduction methods. Despite their appealing theoretical properties, the computational requirements associated with these methods make them impractical for application to large-scale systems of order $10^5$ or higher. Several other methods have been developed that are applicable to large-scale systems, including Krylov-based methods [9,5,7], approximate balanced truncation [11,17], and proper orthogonal decomposition (POD) [14,22,16]. In many cases, this latter group of methods trades computational efficiency for a lack of rigorous guarantees and global error bounds.

A lack of guarantees on the stability of the reduced model is one problem often encountered when using Krylov- or POD-based reduction methods. In particular, a tendency of these methods to produce unstable models has been noted for applications in computational fluid dynamics using the Euler equations [23]. For Krylov-based reduction methods, different approaches have been proposed to resolve the stability issue. The implicit restart algorithm proposed by Grimme et al. [10] guarantees stability by removing the unstable reduced poles through an implicitly restarted Lanczos algorithm. However, the stability comes at the cost that the reduced model is no longer an exact interpolant of the original model, but of a near-by perturbed model. On the other hand, Fourier model reduction (FMR) [23] of Willcox and Megretski preserves stability by first performing a bilinear transformation and then applying model reduction the in the discrete-frequency domain via a truncated Fourier expansion. Similar to FMR, to guarantee stability, Least-squares model reduction method [12] of Gugercin and Antoulas uses a bilinear transformation. Then, model reduction is performed in discrete-time in a different way by combining SVD and Krylov-based methods.

Despite its theoretical properties such as guaranteed stability and a global error bound, and its success in yielding good reduced models [23], the underlying projec-